Statistics

Fall 2021

Lecture 17

Class QZ 16

80 Tickets were sold for $10 each.
One ticket is randomly selected.
the owner of this ticket gets a gift card worth $100.

Find expected value per ticket sold for the fundraisers.

	Net	P(Net)
L1	10 − 100	1/80
	10 − 0	79/80 (L2)

$E.V. = \mu = \bar{x}$

$8.75

Ex:

Consider a binomial prob. dist with 500 trials and .4 prob. of success per trial.

1) $n = 500$ 2) $p = .4$ 3) $q = 1 - p = \boxed{.6}$

4) $\mu = np = \boxed{200}$ 5) $\sigma^2 = npq = \boxed{120}$ 6) $\sigma = \sqrt{\sigma^2} = \sqrt{120} \approx \boxed{11}$

Use empirical rule to find

7) 68% Range

$\mu \pm \sigma$

$= 200 \pm 11$

$\Rightarrow \boxed{189 \text{ to } 211}$

8) Usual Range (95% Range)

$\mu \pm 2\sigma$

$= 200 \pm 2(11)$

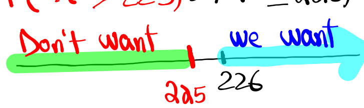
$\Rightarrow \boxed{178 \text{ to } 222}$

Let x be # of successes, find

9) $P(x = 225) = \text{binompdf}(500, .4, 225) = \boxed{.003}$

10) $P(x < 225) = P(x \le 224) = \text{binomcdf}(500, .4, 224) = \boxed{.987}$

11) $P(x > 225) = P(x \ge 226) = 1 - P(x \le 225)$



$= 1 - \text{binomcdf}(500, .4, 225)$

$= \boxed{.010}$

12) $P(178 \le x \le 222) = P(x \le 222) - P(x \le 177)$

$= \text{binomcdf}(500, .4, 222) - \text{binomcdf}(500, .4, 177)$

$= \boxed{.960}$ ✓

Ex:

You are making random guesses on a multiple-choice exam with 60 questions.
Each question has 6 choices with only one correct choice.

1) $n = 60$

2) $p = \frac{1}{6}$

3) $q = \frac{5}{6}$

4) $\mu = np = 60(\frac{1}{6}) = \boxed{10}$

5) $\sigma^2 = npq = 60(\frac{1}{6})(\frac{5}{6}) = \boxed{\frac{25}{3}}$

6) $\sigma = \sqrt{\sigma^2} = \sqrt{\frac{25}{3}} = 2.887 \approx \boxed{3}$

7) 68% Range

$\mu \pm \sigma = 10 \pm 3 \Rightarrow \boxed{7 \text{ to } 13}$

8) 95% Range — Usual Range

$\mu \pm 2\sigma = 10 \pm 2(3) \Rightarrow \boxed{4 \text{ to } 16}$

9) Find P(guessing between 4 and 16, inclusive, correct answers)

$P(4 \le x \le 16) = \text{binomcdf}(60, 1/6, 16) - \text{binomcdf}(60, 1/6, 3) = \boxed{.977}$ ✓

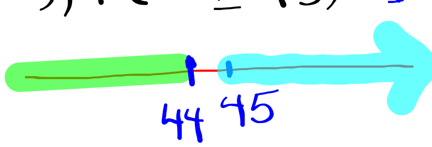
Class QZ 17

Consider a binomial Prob. dist with $n=80$ and $p=.5$. Let x be # of Successes. Find

1) $P(x = 45) = \text{binompdf}(80, .5, 45) = \boxed{.048}$

2) $P(x \le 45) = \text{binomcdf}(80, .5, 45) = \boxed{.891}$

3) $P(x \ge 45) = 1 - \text{binomcdf}(80, .5, 44) = \boxed{.157}$



Geometric Probability

It is very Similar to binomial prob. except there is no fixed # of trials n.

p is prob. of Success for each trial
q is prob. of failure

$p+q=1$, $q=1-p$, p & q remain unchanged for for all trials

x is the # of trials when first Success happens. $x \geq 1$

$P(x) = p \cdot q^{x-1}$

$\mu = \frac{1}{p}$ $\sigma^2 = \frac{q}{p^2}$ $\sigma = \sqrt{\sigma^2}$

Ex: Consider a geometric prob. dist with $p = .2$

$q = 1 - p = .8$

$\mu = \frac{1}{p} = \frac{1}{.2} = \boxed{5}$

$\sigma^2 = \frac{q}{p^2} = \frac{.8}{.2^2} = \boxed{20}$

$\sigma = \sqrt{\sigma^2} = \sqrt{20} \approx \boxed{4.472}$

P(first Success happens on 2nd trial)

$P(x=2) = p \cdot q^{x-1} = .2 \cdot (.8)^{2-1} = .2 \cdot .8^1 = \boxed{.16}$

Now using TI

2nd VARS & Geometpdf(.2, 2) = .16

Prob. that a baseball player hits is .25.

$p = .25$

$q = .75$

$\mu = \frac{1}{p} = \frac{1}{.25} = \boxed{4}$

$\sigma^2 = \frac{q}{p^2} = \frac{.75}{.25^2} = \boxed{12}$

$\sigma = \sqrt{\sigma^2} = \sqrt{12} \approx \boxed{3.464}$

P(first hit is on 3rd attempt) =

$P(x = 3) = p \cdot q^{x-1} = .25(.75)^{3-1} = .25(.75)^2$

$= \boxed{.141}$

$P(x = 3) = \text{geometpdf}(.25, 3) = \boxed{.141}$

P(first hit happens before 4th attempt)

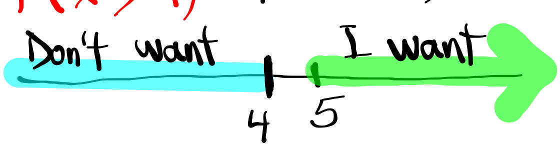
$P(x < 4) = P(x \leq 3)$

$= P(x=3) + P(x=2) + P(x=1)$

$= \text{geometcdf}(.25, 3) = \boxed{.578}$

P(first hit happens after the 4th attempt)

$P(x > 4) = P(x \geq 5) = 1 - P(x \leq 4)$



$= 1 - \text{geometcdf}(.25, 4)$

$= \boxed{.316}$

Poisson Prob. dist

x is # of Successes, $x \geq 0$

Average # of Successes in a fixed interval is given. we can use μ or λ for the average

$\sigma^2 = \mu$, $\sigma = \sqrt{\sigma^2}$

$$P(x) = \frac{\mu^x}{x!} e^{-\mu}, \quad e \approx 2.718$$

Ex:

Consider a poisson prob. dist with $\mu = 4$ in a fixed interval.

$\sigma^2 = \mu$ $\quad \sigma^2 = 4$ $\quad \sigma = \sqrt{\sigma^2} = \sqrt{4} = 2$

68% Range $\Rightarrow$ $\mu \pm \sigma = 4 \pm 2 = \boxed{2 \text{ to } 6}$

$$P(x=5) = \frac{\mu^x}{x!} e^{-\mu} = \frac{4^5}{5!} \cdot e^{-4} \qquad e \approx 2.718$$

(μ or λ)

$= \text{Poissonpdf}(4, 5) = \boxed{.156}$

$P(x \leq 5) = P(x=5) + P(x=4) + P(x=3) + \cdots + P(x=0)$

$= \text{Poissoncdf}(4, 5) = \boxed{.785}$

Suppose Maria gets 9 customers in average per hour.

$\mu = 9$ Fixed interval

Average ✓
Fixed interval ✓
Poisson Prob. dist.

$\sigma^2 = \mu$ $\sigma^2 = 9$

$\sigma = \sqrt{\sigma^2} = \sqrt{9} = 3$

95% Range ⇒ $\mu \pm 2\sigma = 9 \pm 2(3) \Rightarrow$ 3 to 15

P(She gets 12 Customers in a given hr)

$P(x = 12) =$ Poissonpdf$(9, 12) =$ .073

P(She gets fewer than 15 Customers in a given hr)

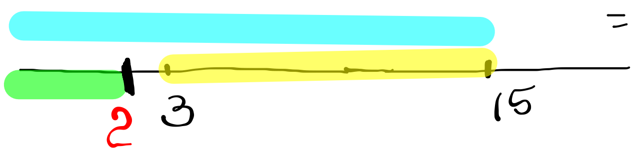
$P(x < 15) = P(x \le 14) =$ Poissoncdf$(9, 14)$

$=$ .959 $\approx 96\%$

P(Maria gets between 3 and 15, inclusive, customers in a given hr)

$P(3 \le x \le 15) = P(x \le 15) - P(x \le 2)$

$=$ Poissoncdf$(9, 15)$ $-$ Poissoncdf$(9, 2)$



$=$ .972 ✓ $\approx 97\%$

SG 17

Let's review more with binomial Prob. dist.

Prob. of Success of certain Surgery is known to be .8.

400 of such Surgeries are randomly Selected.

$n = 400$ $\quad p = .8$ $\quad q = .2$

$\mu = np = 320$ $\quad \sigma^2 = npq = 64$ $\quad \sigma = \sqrt{\sigma^2} = 8$

P(# of successful Surgeries is between 310 and 330, inclusive)

$P(310 \le x \le 330) = \text{binomcdf}(400, .8, 330) - \text{binomcdf}(400, .8, 309)$

$= \boxed{.811} \checkmark \approx 81\%$

Class QZ 18

Consider a binomial prob. dist with $n = 75$ and $p = .6$

Find

$\mu = np = 75(.6) = \boxed{45}$

$\sigma^2 = npq = 75(.6)(.4) = \boxed{18}$

$\sigma = \sqrt{\sigma^2} = \sqrt{18} \approx \boxed{4.243}$